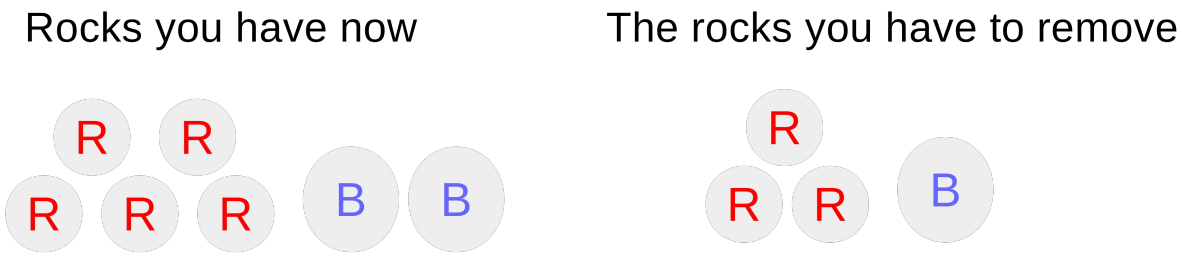
Lesson 3
Polynomial Subtraction

Subtracting polynomials occurs when there is a minus (-) sign in front of a term or between a pair of polynomial quantities. Consider another example with rocks:



How many rocks ***of each type*** do are left? Altogether, there are now 2 red rocks and 1 blue rock. We <u>do not</u> want to say "There are 3 rocks now", because even though it may be true, it doesn't describe the situation completely.

We will do a similar thing when we subtract polynomial expressions, because we will only *subtract like terms*. Consider the next example:

Example 4: Simplify the following polynomial:

$4x^2+6x+8-2x^2-2x-3$

In this example, the task is to simplify the polynomial by combining like terms. Always look at the operations between terms. We see that there is addition <u>and</u> subtraction, so we will have to be careful as we proceed.

First, consider the x^2 terms first: $4x^2$ and $-2x^2$. For these two terms, subtract them, and the result is $2x^2$.

Next, consider the x terms: $+6x$ and $-2x$. Again, the second term is subtraction, so we will combine them using subtraction, and the result is $4x$.

Finally, consider the constants: +8 and –3. This is also a subtraction problem, yielding 5 as a result.

Our solution is each polynomial term together: $2x^2+4x+5$

$4x^2+6x+8-2x^2-2x-3 = 2x^2+4x+5$

Example 5: Simplify the following polynomial:

$-5x^2+3x-8-x^2-7x+10$

In this example, we have more variety with the location of the + and – signs, so we will examine what to do in these instances. Fortunately, we can use our integer property rules to help-- the same rules apply to polynomials.

Note: *remember that even though it isn't written as such, there is a "1" in front of "$-x^2$", and we can imagine it to read "$-1x^2$" We will keep fact in mind as we combine like terms.*

First, consider the x^2 terms: $-5x^2$ and $-x^2$. When we combine integers, we see that $-5-1 = -6$, so similarly, $-5x^2-x^2 = -6x^2$.

Second, consider the x terms: $+3x$ and $-7x$. Notice that $3-7 = -4$, so in the same way, $+3x-7x = -4x$

Finally, we combine the constants. –8 and +10 is pretty easy: $-8+10 = 2$

Consequently, our finally solution is all three polynomial terms: $-6x^2-4x+2$

$-5x^2+3x-8-x^2-7x+10 = -6x^2-4x+2$

Example 6: Simplify the following polynomial:

$(7x^2+6x+4) - (4x^2+2x+1)$

Notice that this example has parentheses, indicating two quantities. To solve this problem, we must <u>subtract</u> the like terms in each quantity from each other. This is because it is *one quantity minus another quantity*: what is inside is important, but

we must think of this as the terms in the first group minus the terms in the seonc group.

Just like in addition of polynomials, the matchup of like terms is still very important. There are a few ways we can look at the problem, so we will solve using a couple of methods, and you can decide which way makes the most sense for you.

$(7x^2+6x+4) - (4x^2+2x+1) = 3x^2+4x+3$

The colors help show which like terms match up with each other, and if you think about moving between terms one at a time (like the colors help show), you get your final answer all on the same line. Remember: you're subtracting.

A different way to solve the same example: if you are comfortable with distribution, this next example may be helpful to you. However, you do not have to simplify polynomials in this way if you do not want to. So, if you would prefer to only combine like terms once per line (similar to the second way of solving the very first addition example), then this next example might be a better illustration for you.

In the very first addition problem, we removed all of the parentheses because it was adding two quantities. Here, *we can remove the parentheses if we* ***distribute the minus sign*** *across all of the second quantity's terms*.

So, to solve $(7x^2+6x+4) - (4x^2+2x+1)$ a little differently, we can have the following steps occur:

$(7x^2+6x+4) - (4x^2+2x+1) = 7x^2+6x+4 -4x^2-2x-1$

$7x^2+6x+4 -4x^2-2x-1 = 3x^2+6x+4-2x-1$

$3x^2+6x+4 -2x-1 = 3x^2+4x+4 -1$

$3x^2+4x+4 -1 = 3x^2+4x+3$

Our final answer of $3x^2+4x+3$ is the same result as before when we left the parentheses in.

Important note: be sure to remember to be careful when subtracting negative numbers-- the same rules for integers apply to polynomials. Consider the following polynomial subtraction problem:

Example 7: Simplify the following: $(3x^2+2x+5) - (-2x^2-6x-3)$

In this problem, every term in the second quantity is negative. Consequently, when we subtract these negative terms from their corresponding like term in the first quantity, the result will be an *increase*.

Example 8:

> You have $8x^2+6x+2$ dollars in your bank account. On a recent shopping trip, you spend $5x^2+2x+3$ dollars using your bank's debit card. Write a polynomial expression showing how much money is in your bank account.

In this question, we again reason through the wording, looking for an indication of what to do with the polynomial expressions. Our main clue word is *spend*, which would indicate *subtraction* as the main operation. Our bank account has $8x^2+6x+2$ dollars in it, so we write that first, and $5x^2+2x+3$ is spent, so that quantity is written second.

$$(8x^2+6x+2) - (5x^2+2x+3) = \mathbf{3x^2+4x-1}$$

The next example is very similar to Example 8, but is written slightly differently.

Example 9:

> On a recent shopping trip, you spend $5x^2+2x+3$ dollars using your bank's debit card. You have $8x^2+6x+2$ dollars in your bank account. Write a

polynomial expression showing how much money is left in your bank account.

This question is worded almost exactly the same, and it is included as a reminder that *order matters when subtracting*. The first quantity written in the problem is $5x^2+2x+3$, and the second is $8x^2+6x+2$, but we have to read carefully to realize that we must write our subtraction problem as the second quantity minus the first quantity. Again, the bank account has $8x^2+6x+2$ dollars in it, so we write that first, and $5x^2+2x+3$ is spent, so that quantity is written second.

$$(8x^2+6x+2) - (5x^2+2x+3) = \mathbf{3x^2+4x-1}$$

We still have the same answer as Example 8 (since the two problems ask the same question), even though the question is worded differently.

Polynomial Subtraction Problem Set

Simplify the following expressions:

$x^2 + 4x - 7 - 3x^2 - 6x + 12$

$-5x^2 + 3x - 8 - x^2 - 7x + 10$

$6x^2 + 9x + 15 - 2x^2 - 7x - 4$

$-9x^2 + x - 4 - 2x^2 - 7x + 7$

$4x^2 + 12x + 11 - 2x^2 - x - 3$

$(3x^2 + 12x) - (2x^2 - x - 3)$

$(5x^2 + 2x + 17) - (5x^2 - 4x - 2)$

A triangle has three sides with the following lengths: $2x - 3$, $4x - 2$, and $3x + 1$. Write a simplified algebraic expression for the perimeter of the triangle.

A large storage container has $(6x^2 + 2x + 9)$ gallons of water in it. A hole was punched in the side, and $(2x^2 + 8x - 7)$ gallons leaked out before the hole was repaired. How much water is in the container now?

A student sees the expression $(4x^2+5x+10) - (2x^2+3x+2)$ and makes the following statement:

> "This problem can be rewritten by just removing the parentheses, and it will mean the same thing."

Is this interpretation correct? Why or why not?

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