Lesson 5
Factoring Polynomials

Factoring polynomials is somewhat different than factoring an integer. When you *factor* a number, you probably already know that you are must write each product. For example,

$$6 = (2)(3)$$

We could have written 6 = (3)(2), but notice that this is the same factorization; it is just in a different order. It is the same factorization because the same numbers are multiplied together.

Some integers have many different factorizations. The number 12, for example, has a few:

$12 = (2)(6)$ $\qquad$ $12 = (3)(4)$ $\qquad$ $12 = (2)(2)(3)$

When we factor polynomials, we must keep this idea in mind: *there are usually many ways to write the products of a number*. The end result of a factored polynomial will always be two binomial quantities multiplied together, looking similar to the material in Lesson 4. *Most exam questions will have polynomial factorizations look the same way.*

AT THIS TIME, PLEASE SKIP AHEAD TO "NUMBER PUZZLE".
RETURN TO THE NEXT PART OF THE LESSON ON THE NEXT PAGE
AFTER YOU HAVE COMPLETED THE NUMBER PUZZLE.

The *Number Puzzle* activity that you completed had you work on a very important skill used in factoring polynomials: think of two numbers that multiply to a certain integer, but add up to another integer. We will use the same idea to factor polynomials.

Consider the following polynomial and its factorization:

$$\boldsymbol{x^2 + 5x + 6 = (x + 2)\,(x + 3)}$$

Where did that factorization come from? Is there a pattern, or a link between any of those numbers?

There certainly is, and to see it, ask yourself this question:

"What does the 2 in $(x + 2)$ and the 3 in $(x + 3)$ have to do with the 5 and 6 in $x^2 + 5x + 6$?"

$$\boldsymbol{x^2 + 5x + 6 = (x + 2)\,(x + 3)}$$

If you are having trouble, think back to the *Number Puzzle* activity, and imagine that the 5 is in the top circle, and the 6 is in the bottom circle. Reasoning through it, the relationship between the numbers is this:

$$(2)(3) = 6 \qquad \text{and} \qquad 2 + 3 = 5$$

Consequently, the two factors of the constant (remember that they must also sum up to the coefficient of the x term!) are the same numbers we put inside the two quantities after x. Since everything is positive in the polynomial, we use addition inside the parentheses.

Note: you can always check that the factorization is correct by multiplying the two quantities together in the same way as Lesson 4. If the result is the same as the polynomial you began with, you have factored it correctly!

Another question to consider: do you think this is always true for polynomial expressions set up like the last example? That is, for any number that in the 5's position is the sum of the two constants in the quantities, and any number in the 6's position is the product of the two constant terms?

Yes, this reasoning is correct, but we will look at another example.

Example 2:

Does it make sense that $x^2 + 7x + 10 = (x + 2)(x + 5)$?

Look carefully: the coefficient for the x term inside the polynomial is the sum of the two quantity constants, and the polynomial's constant is the product.

Your task will always be to factor the polynomial into two quantities. The next example will show a step-by-step process with a "think-aloud" narration of the reasoning behind each step.

Example 3: Factor the polynomial $x^2 + 9x + 20$

Always write the original expression, the equal sign, and then a pair of quantities with only an x written in each one. This is because the goal is *always to find those other two numbers inside the parentheses*. So first, you should write:

$\boldsymbol{x^2 + 9x + 20 = (x \qquad)(x \qquad)}$ **← write this**

Now, when you look at this, start with the polynomial's constant. What does the constant **always** break down into? *The constant is always the product of two numbers*. So, in your head, or written off to the side, you will think of all the ways you can multiply integers to get 20.

20 = (1)(20) or 20 = (2)(10) or 20 = (4)(5) ← write this on the side

Now, consider the coefficient of the x term: *it is always the sum of the constant's product.* Now you will look at your possibilities, which came from the factors of 20. Of those three choices, which pair of numbers adds up to 9?

1 + 20 = 21 or 2 + 10 = 12 or 4 + 5 = 9 ← **write this on the side**

See that 4 + 5 = 9, so that means that 4 and 5 are the two numbers you want to put inside the quantities. It does not matter which number goes in which set of parentheses.

$$x^2 + 9x + 20 = (x \quad 4)(x \quad 5)$$ ← **write this**

Since the numbers 4 and 5 were both positive, you will also write an addition symbol between the variable and constant of each quantity. So, your final answer should look like this:

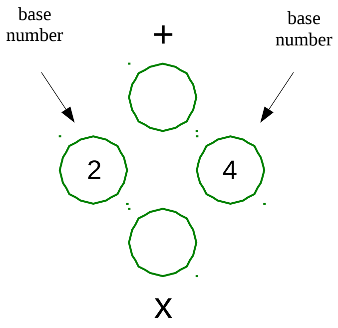
$$x^2 + 9x + 20 = (x + 4)(x + 5)$$ ← **write this**

Complete the problem set and the matching activity at the very end of this lesson.

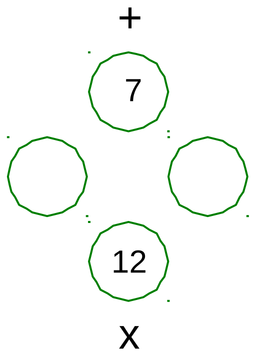
Number Puzzle

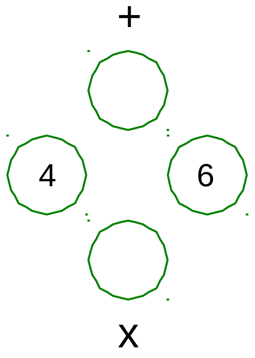
Adapted from CUNY HSE Curriculum 2015

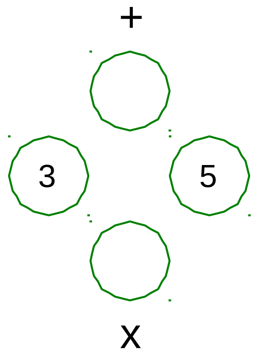
The *Number Puzzle* is a set of four circles.
The circles on the left and right are the two "base" numbers.
The top circle is for the *sum* of the two base numbers.
The bottom circle is for the *product* of the two base numbers.

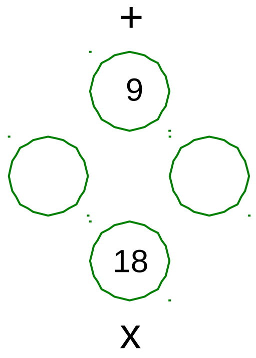


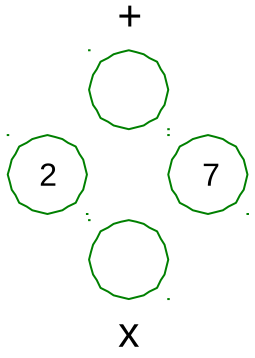
In each *Number Puzzle*, you are given two numbers. Your job is to figure out the missing numbers.

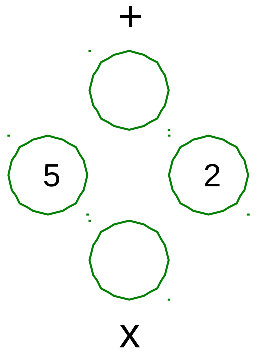


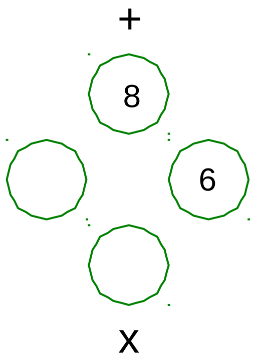








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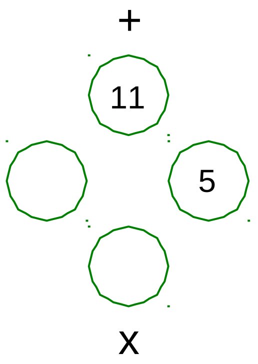
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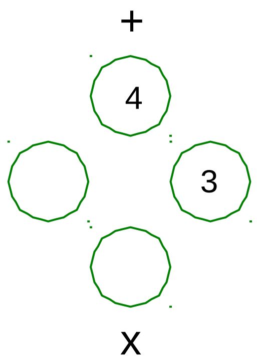
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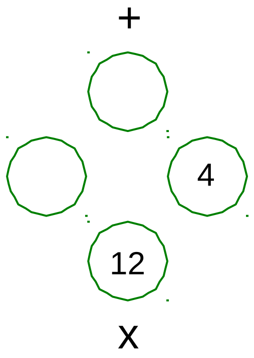
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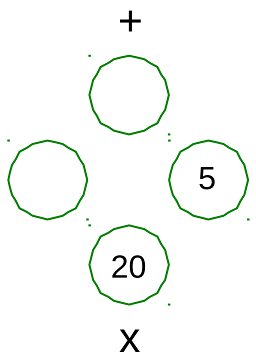
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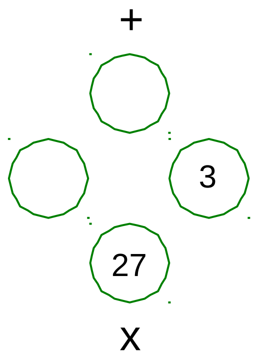
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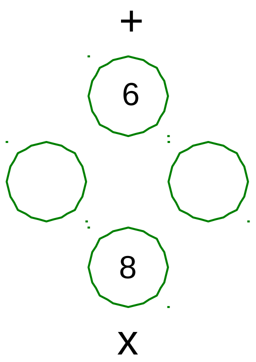
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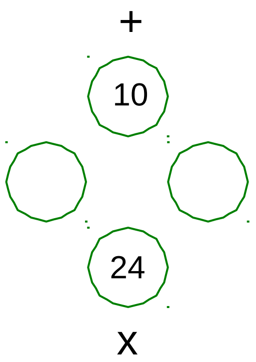
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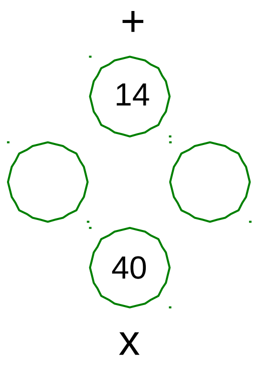
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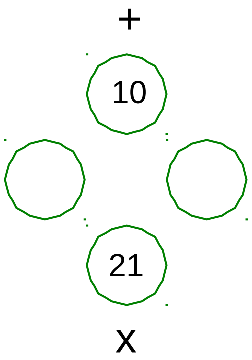
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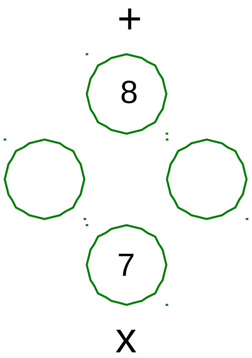
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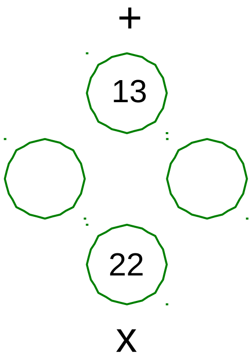
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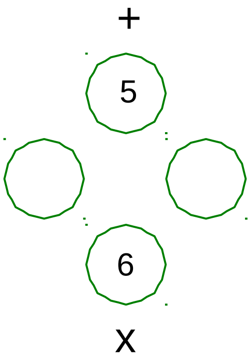
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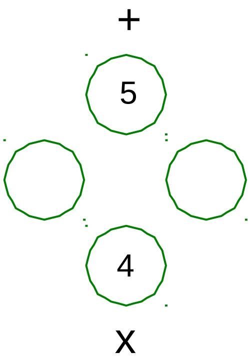
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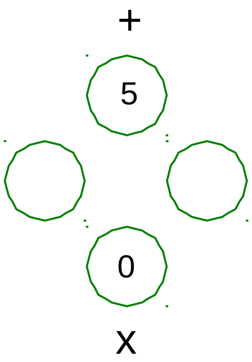
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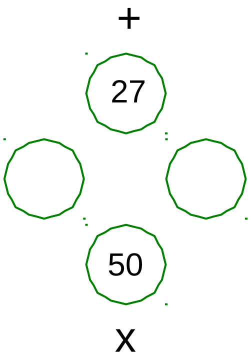
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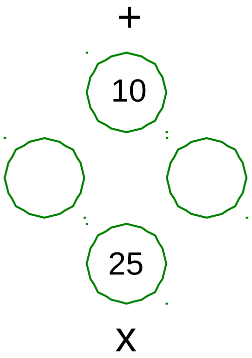
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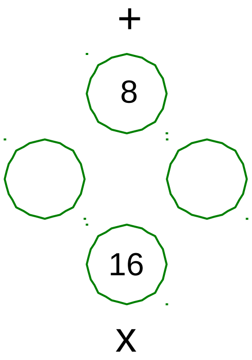
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Problem Set

Factor each polynomial completely

$x^2 + 11x + 18$

$x^2 + 8x + 7$

$x^2 + 6x + 8$

$x^2 + 5x + 4$

$x^2 + 4x + 4$

$x^2 + 7x + 6$

$x^2 + 15x + 26$

$x^2 + 10x + 16$

$x^2 + 10x + 21$

$x^2 + 9x + 20$

$x^2 + 6x + 7$

$x^2 + 7x + 12$

Cut the squares apart, and mix up the pieces.
Match equivalent expressions.
This 4 X 4 square should be the result.

		$x^2-4x-12$	$(x+2)(x-6)$		x^2-16	$(x+4)(x-4)$		$x^2+7x+10$	$(x+2)(x+5)$		
	$(x-2)(x+2)$			$x^2-10x+24$			$(x+3)(x+3)$			$(x+1)(x-1)$	
	x^2-4			$(x-4)(x-6)$			x^2+6x+9			x^2-1	
		$x^2+3x-18$	$(x-3)(x+6)$		$x^2+6x-16$	$(x-2)(x+8)$		$x^2+7x-18$	$(x+9)(x-2)$		
	$(x+2)(x+3)$			x^2-9			$(x-2)(x+3)$			$x^2-7x+12$	
	x^2+5x+6			$(x+3)(x-3)$			x^2-x-6			$(x-4)(x-3)$	
		$(x^2-6x-16)$	$(x+2)(x-8)$		$x^2-2x-15$	$(x+3)(x-5)$		$(x-2)(x-9)$	$x^2-11x+18$		
	x^2+4x+3			$(x+3)(x+1)$			$x^2+3x-10$			$x^2-8x+16$	
	$(x+3)(x+1)$			x^2+4x+3			$(x-2)(x+5)$			$(x-4)(x-4)$	
		x^2+4x+4	$(x+2)(x+2)$		$(x+2)(x+4)$	x^2+6x+8		x^2-x-12	$(x+3)(x-4)$		