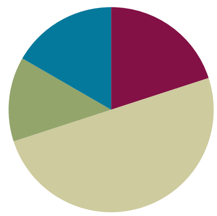
Lesson 10

Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Divide with Number Disks **4.NBT.1** (4 minutes)
- Group Count by 90° **4.MD.7** (1 minute)
- Break Apart 90, 180, and 360 **4.MD.7** (4 minutes)
- Physiometry **4.G.1** (3 minutes)

Divide with Number Disks (4 minutes)

Materials: (S) Personal white board

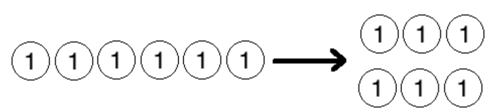
Note: This fluency activity reviews Module 3 content.

T: (Display 6 ÷ 2.) On your personal white boards, draw number disks to represent the expression.

S: (Draw 6 ones disks, and divide them into 2 groups of 3.)

T: Say the division sentence in unit form.

S: 6 ones ÷ 2 = 3 ones.



Continue with the following possible sequence: 60 ÷ 2; 600 ÷ 2; 6,000 ÷ 2; 9 tens ÷ 3; 12 tens ÷ 4; and 12 tens ÷ 3.

Group Count by 90° (1 minute)

Note: If students struggle to connect counting groups of 9, groups of 9 tens, and groups of 90, write the counting progressions on the board.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Nines to 36
- 9 tens to 36 tens
- 90 to 360
- 90° to 360° (while turning)

Break Apart 90, 180, and 360 (4 minutes)

Materials: (S) Personal white board

Note: This fluency exercise prepares students for unknown angle problems in Lessons 10 and 11.

T: (Project a number bond with a whole of 90. Fill in 45 for one of the parts.) On your personal white boards, write the number bond, filling in the unknown part.

Continue to break apart 90 with the following possible sequence: 35, 25, 65, and 15.

T: (Project a number bond with a whole of 180. Fill in 170 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

Continue to break apart 180 with the following possible sequence: 90, 85, 45, and 125.

T: (Project a number bond with a whole of 360. Fill in 180 for one of the parts.) On your boards, write the number bond, filling in the unknown part.

Continue to break apart 360 with the following possible sequence: 90, 45, 270, 240, and 315.

Physiometry (3 minutes)

Note: Kinesthetic memory is strong memory. This fluency activity reviews terms from Lessons 1–8.

T: Stand up. (Students stand and follow the series of directions below.)
T: Model a 90° angle with your arms.
T: Model a 180° angle with your arms.
T: Model a 270° angle.
T: Model a 360° angle.
T: Point to the walls that run perpendicular to the back of the room.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 90° to your left.
T: Turn 180°.

T: Turn 90° to your left.

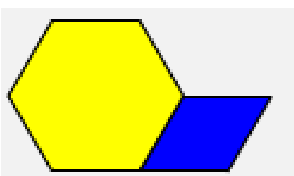
T: Turn 180°.

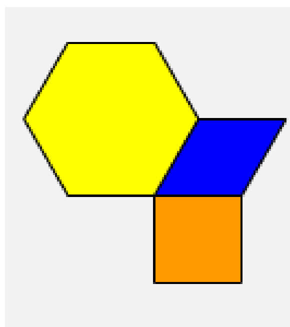
T: Turn 270° to your right.

T: Turn 180° to your left.

Application Problem (8 minutes)

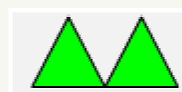
Using pattern blocks of the same shape or different shapes, construct a straight angle. Which shapes did you use? Compare your representation to that of your partner. Are they the same? Which pattern block can you add to your existing shape to create a 270° angle? How can you tell?





NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Seeking to construct a straight angle, some students may place two triangles or trapezoids side by side, leaving gaps between the angle sides. Encourage them to verify 180° by adding the interior angles of the pattern blocks. Ask, "What shape could fit in this gap? How can you confirm that you've made a straight angle?"



Note: This Application Problem builds from the previous lesson, where students found the angle measures of pattern blocks and verified the measures with a protractor. In this Application Problem, students use pattern blocks to form a straight angle, and then examine the relationship of the parts to the whole and discover that there are many ways to compose and decompose a straight angle. This leads into today's Concept Development, where students deepen their understanding of the additive nature of angle measure.

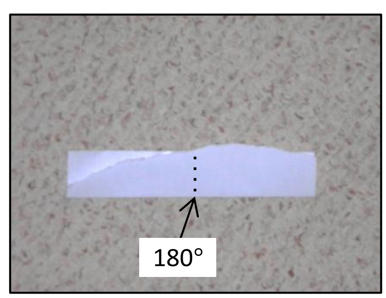
Concept Development (30 minutes)

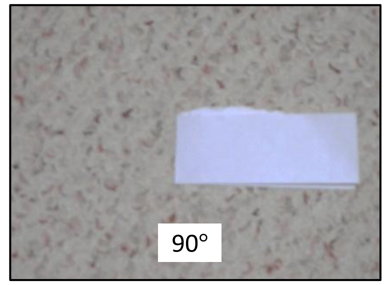
Materials: (T/S) Blank paper (full sheet of letter-size paper ripped into two pieces), personal white board, straightedge, protractor, pattern blocks

Problem 1: Use benchmark angle measures to show that angle measures are additive.

T: Grab a blank sheet of ripped paper. Fold it in half from bottom to top. Fold it from left to right. Open the paper back up one fold. (Demonstrate.) Run your finger along the line of the horizontal fold. Consider the fold. Mark the vertex with a dot. What special angle have you created?

S: A straight angle. → Its measurement is 180°.




90°

T: Fold your paper back left to right. Be sure it is folded so that the previously folded edge is directly on top of itself. Run your finger along the folded sides. What angle have you created now, if the vertex of the angle is at the corner of the folds?

S: A right angle. → A 90° angle.

T: Fold the vertical side down to match up with the horizontal side, like this. (Demonstrate.) Unfold. How many angles has the right angle been decomposed into?

S: Two!

T: What do you notice about the two angles?

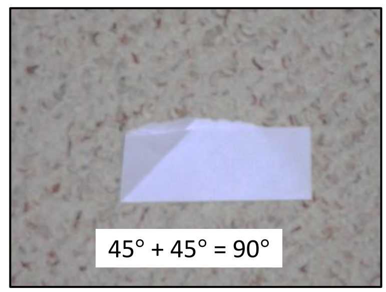
S: They are the same.

T: How can you tell?

S: One angle fits exactly on top of the other.

T: Discuss with your partner. How can you determine the measurement of each angle?

S: We can take 90° and divide it by two. → We can think of what number plus itself equals 90. It's 45. → We could use a protractor to measure.


45° + 45° = 90°

T: Unfold your paper one fold.

T: Let's look at the angles. What do we see?

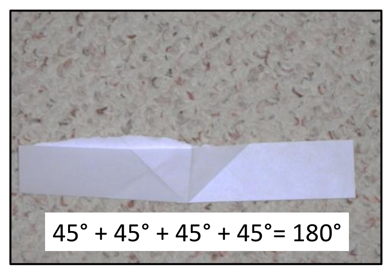
S: We see the two 45° angles.

T: Say the number sentence that shows the total of the angle measurements.

S: 45° + 45° = 90°.

T: Unfold another fold. What do you see now?

S: Four angles.

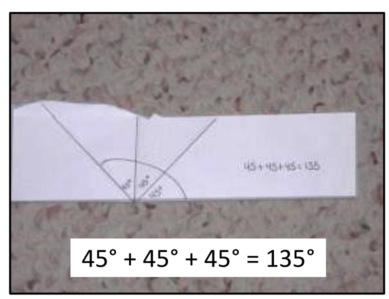

45° + 45° + 45° + 45°= 180°

T: What do you notice?

S: They are all the same. I can tell because if I fold the paper, they stack evenly on top of each other.

T: Say the number sentence that shows the total of the angle measurements.

S: 45° + 45° + 45° + 45° = 180°. → That makes sense because we have a straight line along this side.


45° + 45° + 45° = 135°

T: What if we just looked at three of the angles? Draw an arc on your paper to show the angle created by looking at three of the angles together. Say the number sentence that shows the total of the angle measurements.

S: 45° + 45° + 45° = 135°. → 180° – 45° = 135°.

T: Let's verify with a protractor. Use your straightedge to trace along each crease. Measure and label each angle measure, and then measure and label the entire angle. Write the number sentence.

Students measure, label, and write the number sentence.

Problem 2: Demonstrate that the angle measure of the whole is the sum of the angle measures of the parts.

T: Fold a different ripped piece of paper to form a 90° angle as we did before.

T: Fold the upper left-hand section of your paper down. This time, the corner should not meet the bottom of your paper. (Demonstrate.)

T: Open the fold that you just created. What do you see?

S: I see two angles. They are not the same size.

T: Compare your angles to your partner's. Are they the same?

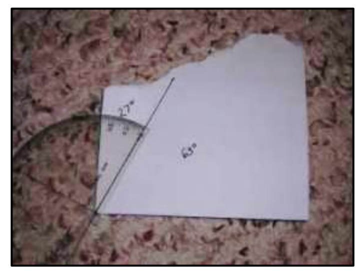
S: No, they look different.

T: Why is that?

S: We each folded our paper differently.

T: Follow these directions:

1. Use a straightedge to draw a segment on the fold.
2. Measure the two angles with your protractor.
3. Label each angle measure.
4. Write the number sentence to show the sum of the two angles.

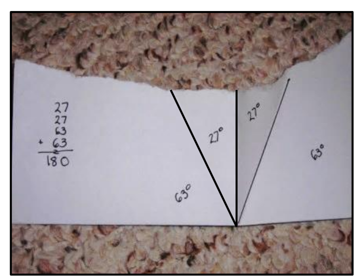


T: (Allow students time to work.) What do you notice?

S: The angles added up to 90°. 63° + 27° = 90°. That shows the whole! → Mine didn't add up to 90°. They added up to 88°. → Mine added up to 91°. That doesn't make sense because the angle we started with was 90°. If we split it into two parts, the parts should add up to the whole. It's just like when we add or subtract numbers. I must have measured the angles wrong. Let me try again!

T: Unfold your paper another time. What do you see?

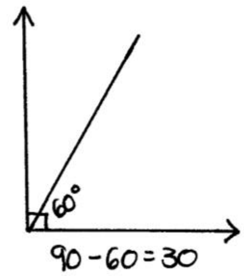
S: There are four angles instead of two. → These four angles combine to make a straight angle.



T: Repeat the same process with these four angles to find their sum. Do you need to measure all of the angles?

S: No. I know their measurements because when I folded the paper, I was making angles that are the same. One unknown angle is 27° and the other is 63°. The angles add together to make a measurement of 180°. That makes sense because the paper has a straight edge. When we fold the paper, we are splitting the original angle into parts. All of those parts have to add up to the original angle because the whole part doesn't change when we fold it. It stays the same.

Problem 3: Given the angle measure of the whole, find the unknown measure of the part. Write an equation using a symbol for the unknown angle measure.



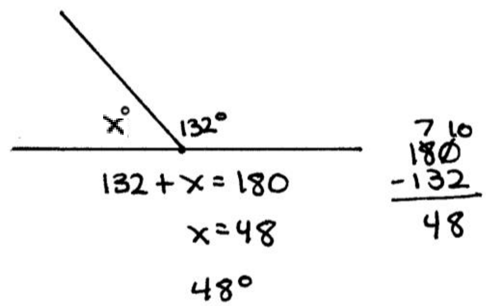
T: (Using a protractor, construct a 90° angle on the board. Within that angle, measure and label a 60° angle.)

T: Discuss with your partner how we can find the measurement of the unknown angle. Use what we just learned.

S: I know that the measurement of the large angle is 90°. If one of the parts is 60°, I can figure out the other part by subtracting 60 from 90. 90 – 60 = 30. The other angle is 30°. → I know that 60 + 30 is 90, so the other angle must be 30°.

MP.2

T: When we take a whole angle and break it into two parts, if we know the angle measurement of one part, we can find the angle measurement of the other part by subtracting.



T: (Draw a straight angle on the board. Use a protractor to draw a 132° angle. Label the angle as 132°. Indicate that we know the 180° measure and the 132° measure, but that we do not know the measure of x.)

T: Work with your partner to find the unknown angle.

S: If the straight angle measures 180° and one part is 132°, then the other angle must be 48° because 180 – 132 = 48. → I solved it because I know that 132 + 48 = 180 by counting on. → I knew that 130 plus 50 is 180, and 50 minus 2 is 48.

T: Let's write an equation and use x to represent the measure of the unknown angle. Let's start with the known part. What is the known part?

S: 132.

T: What is the total?

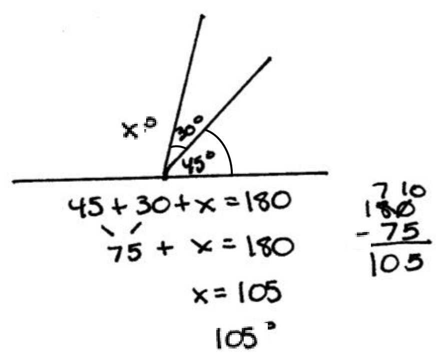
S: 180.

T: Say the equation. Start with the known part.

S: 132 + x = 180.

T: Say it in a subtraction sentence starting with the whole.

S: 180 – 132 = x.



T: (Draw a straight angle on the board. Using a protractor, measure a 75° angle. Then, using a protractor, subdivide the angle into a 45° angle and a 30° angle.)

T: What is different about this angle than the angles that we have been working with?

S: The angle is split into three parts instead of two.

T: How can we solve for the unknown angle? Write the equation.

S: 45 + 30 + x = 180. → This is just like when we have to find the unknown part when we add numbers. We find the sum of the two angles that we know, and then we subtract from the total. 45 + 30 = 75. 180 – 75 = 105.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

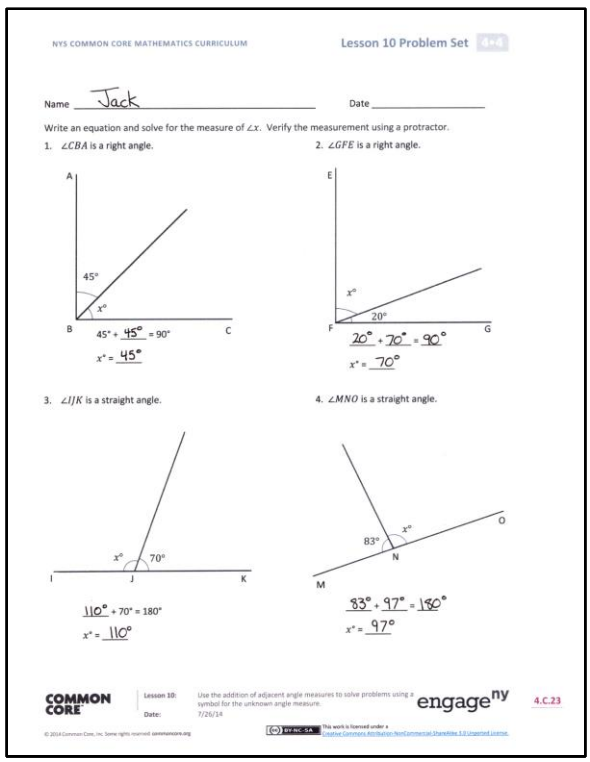
Students working below grade level and others may benefit from additional scaffolding of Page 1 of the first page of the Lesson 10 Problem Set. It may be helpful to include a subtraction sentence frame for solving for x. For example, in Problem 1, provide 90 – 45 = x. Build student independence gradually. Have students become confident with writing their own subtraction sentence after a few examples. Then, for the final problems, encourage students to subtract mentally.

Student Debrief (10 minutes)

Lesson Objective: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Problem Set 4•4

Name Jack Date

Write an equation and solve for the measure of ∠x. Verify the measurement using a protractor.

1. ∠CBA is a right angle.

45° + 45° = 90°

x° = 45°

2. ∠GFE is a right angle.

20° + 70° = 90°

x° = 70°

3. ∠IJK is a straight angle.

110° + 70° = 180°

x° = 110°

4. ∠MNO is a straight angle.

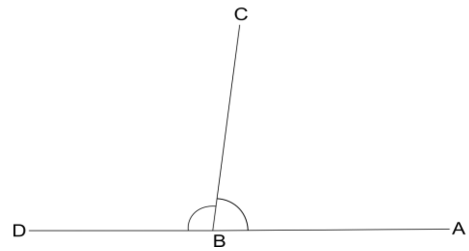
83° + 97° = 180°

x° = 97°

Any combination of the questions below may be used to lead the discussion.

- For Problems 1–6, why is it important to know that we are starting with a right angle or a straight angle?
- For Problem 7, why is it important to know that ACDE is a rectangle?
- Why is it important to be precise when measuring angles?
- When two angles add to 90°, we say that they are **complementary angles**. When two angles add to 180°, we say that they are **supplementary angles**. What examples did we have of complementary angles? Of supplementary angles?

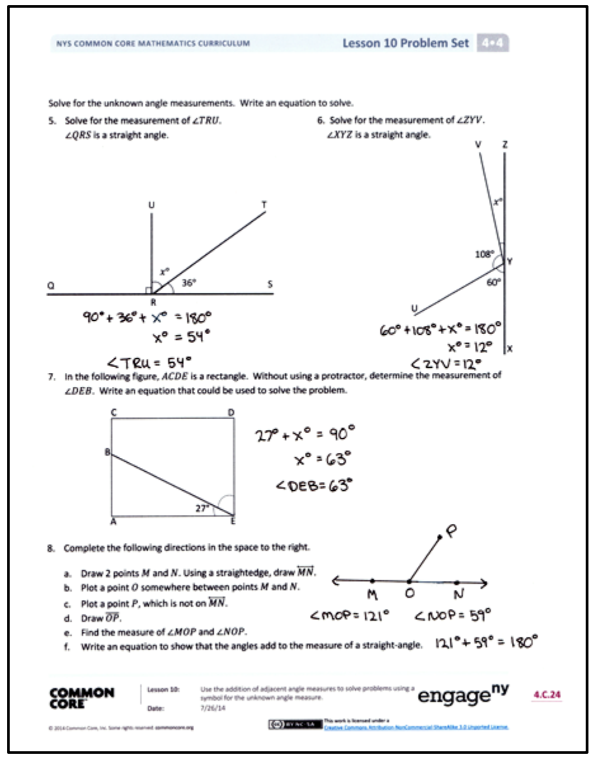
- (Write ∠ABC + ∠CBD = 180°.) When two or more angles meet to form a straight line, we saw that the angle measures add up to 180° (as shown to the right). As we saw yesterday, the angle symbol with an *s* just means *angles*. It's the plural of *angle*. (Write ∠*s* on a line.) ∠*s* on a line translates as "we have angles that together add up to make a line." How can we use the sum of angles on a line being 180° to solve problems?
- What new (or significant) math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?



∠s on a line:
$\angle ABC + \angle CBD = 180°$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Problem Set 4•4

Solve for the unknown angle measurements. Write an equation to solve.

5. Solve for the measurement of $\angle TRU$.
   $\angle QRS$ is a straight angle.

   90° + 36° + x° = 180°
   x° = 54°
   ∠TRU = 54°

6. Solve for the measurement of $\angle ZYV$.
   $\angle XYZ$ is a straight angle.

   60° + 108° + x° = 180°
   x° = 12°
   ∠ZYV = 12°

7. In the following figure, $ACDE$ is a rectangle. Without using a protractor, determine the measurement of $\angle DEB$. Write an equation that could be used to solve the problem.

   27° + x° = 90°
   x° = 63°
   ∠DEB = 63°

8. Complete the following directions in the space to the right.
   a. Draw 2 points M and N. Using a straightedge, draw $\overleftrightarrow{MN}$.
   b. Plot a point O somewhere between points M and N.
   c. Plot a point P, which is not on $\overleftrightarrow{MN}$.
   d. Draw $\overrightarrow{OP}$.
   e. Find the measure of $\angle MOP$ and $\angle NOP$.
   f. Write an equation to show that the angles add to the measure of a straight-angle. 121° + 59° = 180°

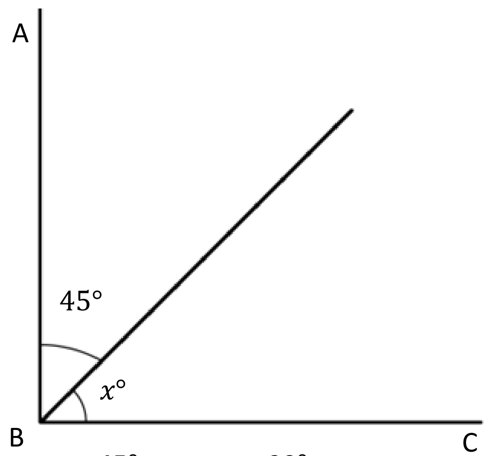
   ∠MOP = 121° ∠NOP = 59°

COMMON CORE Lesson 10: Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure. Date: 7/26/14 engage^ny 4.C.24

Name ____ Date ____________________

Write an equation, and solve for the measure of $\angle x$. Verify the measurement using a protractor.

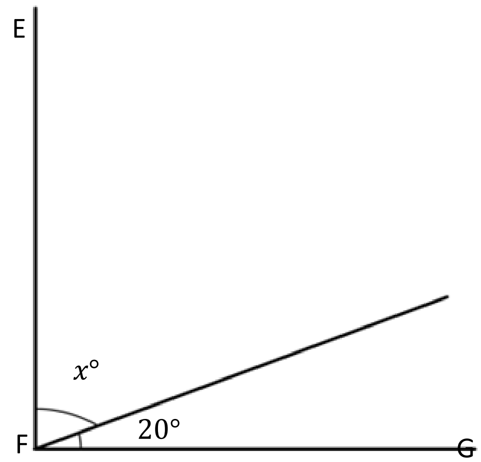
1. $\angle CBA$ is a right angle.



45° + _____ = 90°

$x°$ = _____

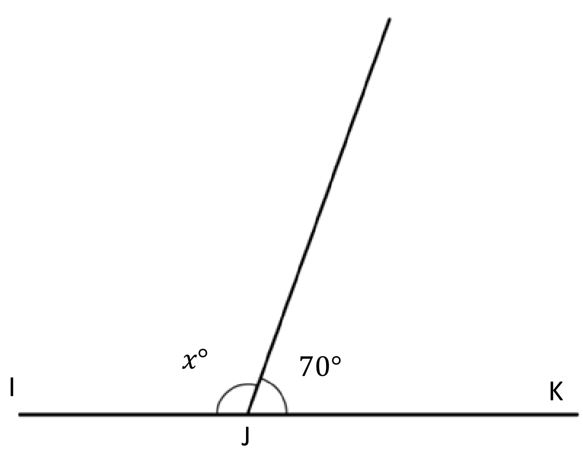
2. $\angle GFE$ is a right angle.



_____ + _____ = ____

$x°$ = _____

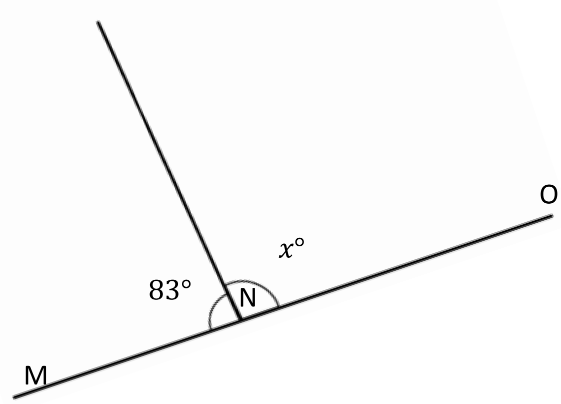
3. $\angle IJK$ is a straight angle.



_____ + 70° = 180°

$x°$ = _____

4. $\angle MNO$ is a straight angle.

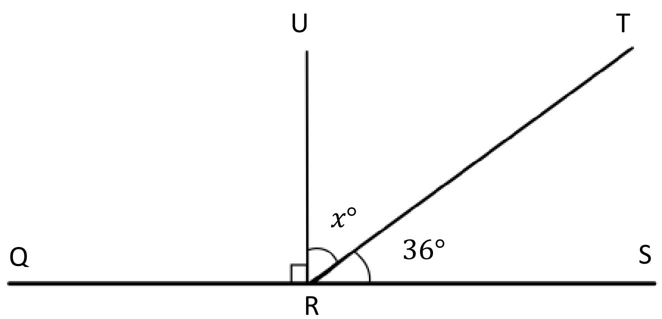


_____ + _____ = ____

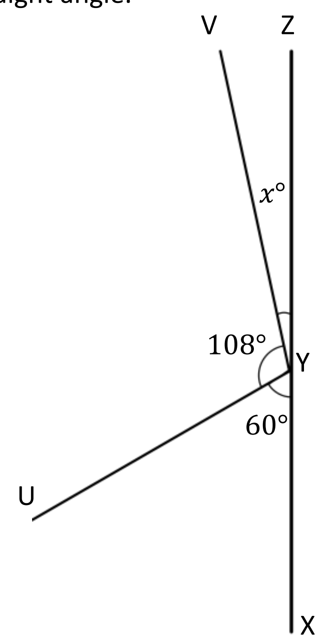
$x°$ = _____

Solve for the unknown angle measurements. Write an equation to solve.

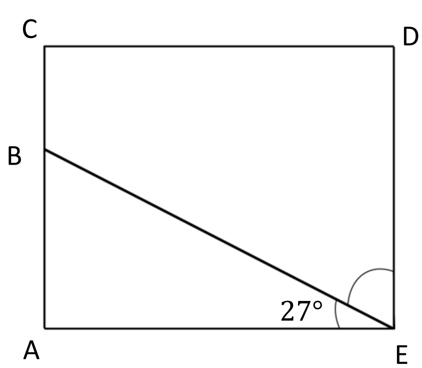
5. Solve for the measurement of $\angle TRU$.
   $\angle QRS$ is a straight angle.



6. Solve for the measurement of $\angle ZYV$.
   $\angle XYZ$ is a straight angle.



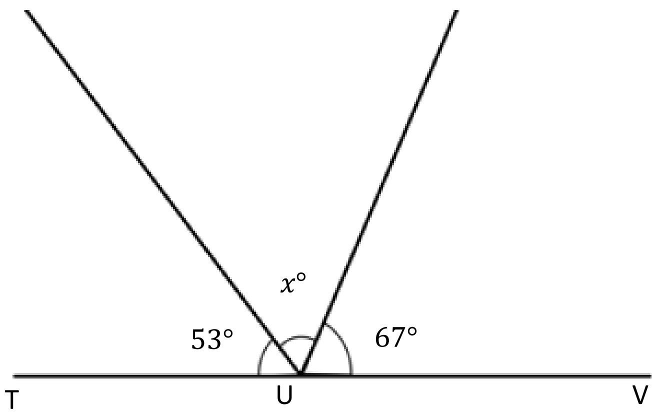
7. In the following figure, $ACDE$ is a rectangle. Without using a protractor, determine the measurement of $\angle DEB$. Write an equation that could be used to solve the problem.



8. Complete the following directions in the space to the right.
   a. Draw 2 points: M and N. Using a straightedge, draw $\overleftrightarrow{MN}$.
   b. Plot a point O somewhere between points M and N.
   c. Plot a point P, which is not on $\overleftrightarrow{MN}$.
   d. Draw $\overline{OP}$.
   e. Find the measure of $\angle MOP$ and $\angle NOP$.
   f. Write an equation to show that the angles add to the measure of a straight angle.

Name ______________________________ Date ________________

Write an equation, and solve for x. $\angle TUV$ is a straight angle.



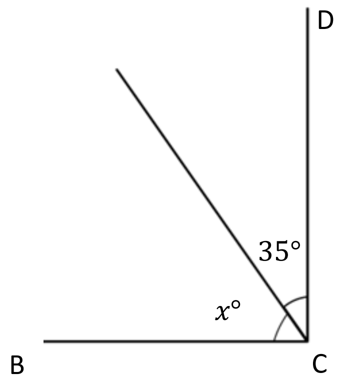
Equation: ____

$x° =$ ____________

Name ____ Date ______________________

Write an equation, and solve for the measurement of $\angle x$. Verify the measurement using a protractor.

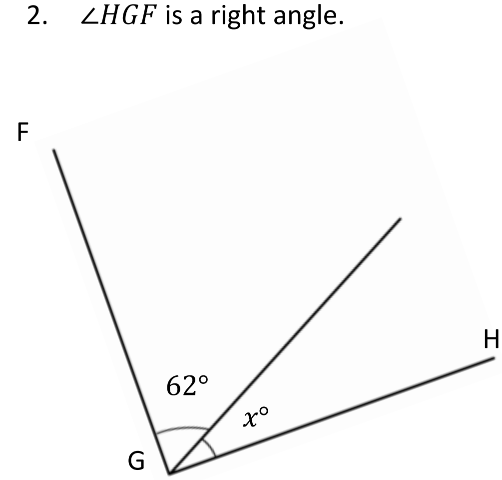
1. $\angle DCB$ is a right angle.



______ + 35° = 90°

$x°$ = ______

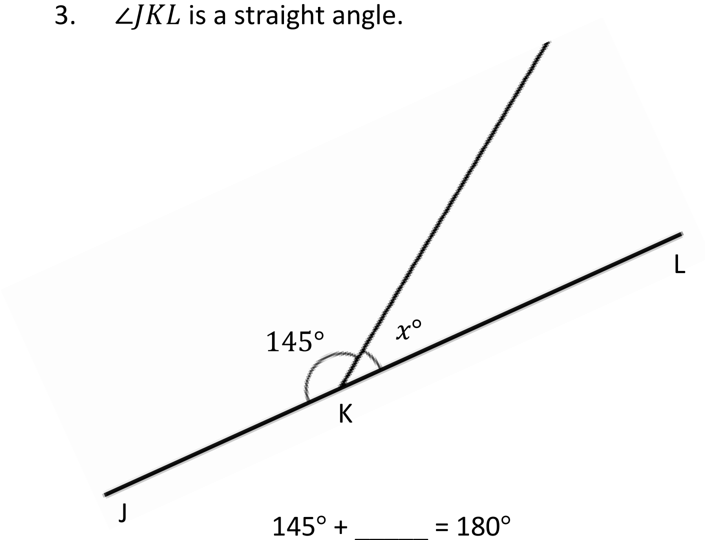
2. $\angle HGF$ is a right angle.



______ + ______ = _____

$x°$ = ______

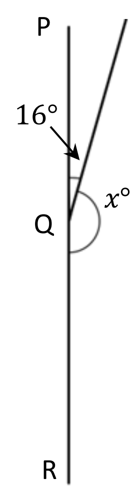
3. $\angle JKL$ is a straight angle.



145° + ______ = 180°

$x°$ = ______

4. $\angle PQR$ is a straight angle.

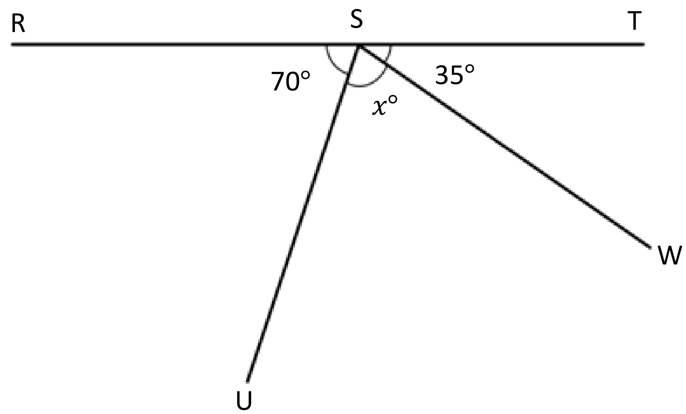


______ + ______ = _____

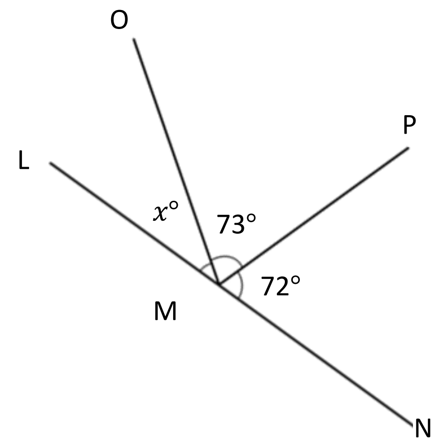
$x°$ = ______

Write an equation, and solve for the unknown angle measurements.

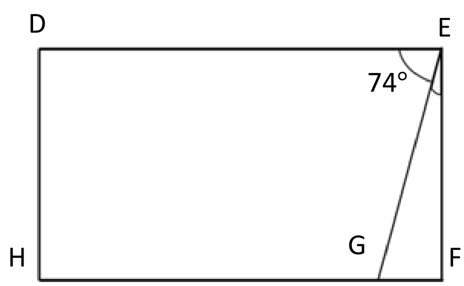
5. Solve for the measurement of $\angle USW$.
$\angle RST$ is a straight angle.



6. Solve for the measurement of $\angle OML$.
$\angle LMN$ is a straight angle.



7. In the following figure, $DEFH$ is a rectangle. Without using a protractor, determine the measurement of $\angle GEF$. Write an equation that could be used to solve the problem.



8. Complete the following directions in the space to the right.

a. Draw 2 points: Q and R. Using a straightedge, draw $\overleftrightarrow{QR}$.
b. Plot a point S somewhere between points Q and R.
c. Plot a point T, which is not on $\overleftrightarrow{QR}$.
d. Draw $\overline{TS}$.
e. Find the measure of $\angle QST$ and $\angle RST$.
f. Write an equation to show that the angles add to the measure of a straight angle.