A-CED Rewriting equations

Alignments to Content Standards: A-CED.A.4

Task

In each of the equations below, rewrite the equation, solving for the indicated variable

a. If F denotes a temperature in degrees Fahrenheit and C is the same temperature measured in degrees Celsius, then F and C are related by the equation

$$F = \frac{9}{5}C + 32.$$

Rewrite this expression to solve for C in terms of F.

b. The surface area S of a sphere of radius r is given by

$$S = 4\pi r^2.$$

Solve for r in terms of S.

c. The height h of a diver over the water is modeled by the equation

$$h = -5t^2 + 8t + 3$$

where h denotes the height of the diver over the water (in meters) and t is time measured in seconds. Rewrite this equation, finding t in terms of h.

d. A bacteria population P is modeled by the equation

$$P = P_0 10^{kt}$$

where time t is measured in hours, k is a positive constant, and P_0 is the bacteria population at the beginning of the experiment. Rewrite this equation to find t in terms

of P.

IM Commentary

The goal of this task is to manipulate equations in order to solve for a specified variable. For each equation, the teacher may wish to prompt students to put into words what the original equation and rewritten equations say. In addition, the contexts have been chosen so that it is possible to motivate why we might be interested in highlighting the requested quantity. For (a), the desired equation converts from degrees Fahrenheit to degrees Celsius. For (b), the new expression gives the radius of the sphere in terms of its surface area. For (c), the rewritten equation tells us at what time(s) the diver is at a specified height over the water. For (d), the equation tells us at what time the bacteria population took a given value.

Solution

a. To solve for C in terms of F we can subtract 32 from both sides and then multiply both sides by $\frac{5}{9}$ giving

$$C = \frac{5}{9}F - \frac{160}{9}.$$

b. Here we divide both sides by 2π and then take the square root of each side. Note that normally $\sqrt{r^2}$ could be r or $-r$ but in this context, the radius can only be positive and so the value $-r$ can be discarded:

$$r = \sqrt{\frac{S}{4\pi}}.$$

c. We can rewrite the equation as

$$-5t^2 + 8t + 3 - h = 0.$$

This equation can then be solved using the quadratic formula and we find

$$t = \frac{-8 \pm \sqrt{64 + 20(3-h)}}{-10}.$$

Within the context of the problem h and t must be non-negative. The fact that there are often *two* values of time for certain heights h represents the fact that the diver hits those heights twice, first on the way up, and then again on the way down.

d. In order to solve for t we will need to take a logarithm. The log will be of base 10 since the base of the exponential is 10. First dividing both sides by P_0 and then applying the log gives

$$\log_{10}\left(\frac{P}{P_0}\right) = kt.$$

This means that

$$t = \frac{\log_{10}\left(\frac{P}{P_0}\right)}{k}.$$