Illustrative Mathematics

A-SSE Animal Populations

Alignments to Content Standards: A-SSE.A.1.a A-SSE.A.1 A-SSE.A.2

Task

Suppose P and Q give the sizes of two different animal populations, where $Q > P$. In (a)–(f), say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations.

a. $P + Q$ and $2P$

b. $\dfrac{P}{P+Q}$ and $\dfrac{P+Q}{2}$

c. $(Q-P)/2$ and $Q - P/2$

d. $P + 50t$ and $Q + 50t$

e. $\dfrac{P}{P+Q}$ and 0.5

f. $\dfrac{P}{Q}$ and $\dfrac{Q}{P}$

IM Commentary

In this task students have to interpret expressions involving two variables in the context of a real world situation. All given expressions can be interpreted as quantities that one might study when looking at two animal populations. For example, $\frac{P}{P+Q}$ is the fraction that population P makes up of the combined population $P+Q$.

Although the context is quite thin, posing the question in terms of populations rather than bare numbers encourages students to think about the variables as numbers and provides avenues for them to use their common sense in explaining their reasoning. This encourages them to see expressions as having meaning in terms of operations, rather than seeing them as abstract arrangements of symbols.

(Task adapted from *Algebra: Form and Function, McCallum et al., Wiley 2010*. Solutions for (e)-(f) were written by Ashley Trujillo , Benjamin Rivera, James Stoddard, Jian Feng Huang, Lisa Brown, Lisa Orton, Moses Baca, Ruth Ramirez, and Saul Goldblatt.)

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Solution

a. The expression $P+Q$ is larger.

- The expression $P+Q$ gives the total size of the two populations put together.
- The expression $2P$ gives the size of a population twice as large as P.
- Putting the smaller population together with the larger yields more animals than merely doubling the smaller.

Another way to see this is to notice that $2P=P+P$, which is smaller than $P+Q$ because adding P to P is less than adding Q to P.

b. The expression $\frac{P+Q}{2}$ is larger.

- The total size of the two populations put together is $P+Q$, so the expression $\frac{P}{P+Q}$ gives the fraction of this total belonging to P. Since $P<P+Q$, this will be a number less than 1. For instance, if $P=100$ and $Q=150$, this fraction equals $100/(100+150)=0.4=40\%$.
- The average or mean size of the two populations is their sum divided by two, or $\frac{P+Q}{2}$. This will be a number between P and Q, so it is larger than 1 (since P and Q describe animal populations). For instance, if $P=100$ and $Q=150$, the average is $(100+150)/2=125$.

c. The expression $Q-P/2$ is larger.

- The expression $(Q-P)/2$ gives half the difference between P and Q. For instance, if $Q=150$ and $P=100$, half the difference is $(150-100)/2=25$.

- The expression $Q - P/2$ gives the difference between Q and a population half the size of P. For instance, if $Q = 150$ and $P = 100$, this difference equals $150 - 100/2 = 100$.

To see why the second of these is bigger, write

$$(Q - P)/2 = Q/2 - P/2$$

In the expression $Q - P/2$, we subtract $P/2$ from Q. But in $(Q - P)/2$, we subtract the same value, $P/2$, from a smaller amount, $Q/2$.

d. The expression $Q + 50t$ is larger.

- In both expressions, the same value, $50t$, is added to the population.
- Since $P < Q$, adding $50t$ to P results in a smaller value than adding the same amount to Q.

e. The expression 0.5 is larger.

- The total size of the two populations put together is $P + Q$, so the expression $\frac{P}{P+Q}$ gives the fraction of this total population belonging to P. Since there are fewer animals in population P than Q, this fraction is less than $\frac{1}{2}$. For instance, if $P = 100$ and $Q = 150$, this fraction equals $100/(100 + 150) = 0.4$.
- $\frac{P}{Q}$ and $\frac{Q}{P}$ can be interpreted in two different ways.

  - $\frac{P}{Q}$ can be interpreted as a unit rate, namely, the number of animals in population P for every 1 animal in population Q. Similarly, $\frac{Q}{P}$ can be interpreted as the number of animals in population Q for every 1 animal in population P. Since there are more animals in population Q, the unit rate $\frac{Q}{P}$ will be greater than the unit rate $\frac{P}{Q}$.

    For example, if $P = 100$ and $Q = 150$, then $\frac{100}{150} = \frac{2}{3}$, so there would be $\frac{2}{3}$ of an animal in population P for every 1 animal in population Q, while $\frac{150}{100} = \frac{3}{2}$, so there would be $\frac{3}{2}$ of an animal in population Q for every 1 animal in population P.
    Some people think it is awkward to talk about fractions of animals, so here is another way to think about it:

- $\frac{P}{Q}$ can also be interpreted as the fraction that population P is of population Q.

Since there are fewer animals in population P, as a fraction of the population of Q it will be less than 1. Similarly, $\frac{Q}{P}$ can also be interpreted as the fraction that population Q is of population P. Since there are more animals in population Q, as a fraction of the population of P it will be greater than 1.

For example, if $P = 100$ and $Q = 150$, this fraction equals $\frac{100}{150} = \frac{2}{3}$, so there are $\frac{2}{3}$ as many animals in population P as there are in population Q, while $\frac{150}{100} = \frac{3}{2}$, so there are $\frac{3}{2}$ as many animals in population Q as there are in population P.