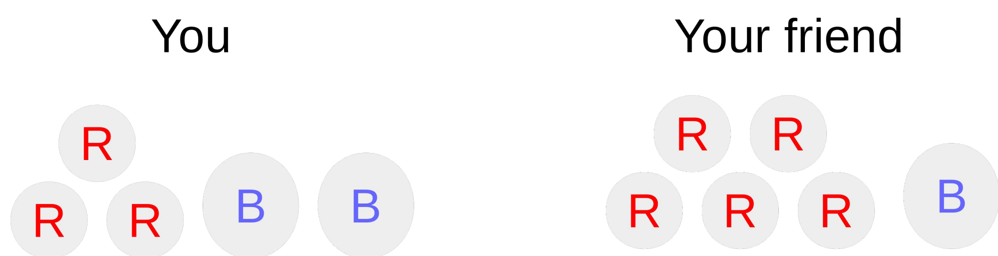
Lesson 2
Adding Polynomials

Adding algebraic expressions is an important component of higher-order algebra skills. Addition is the most basic method of combining the two or more parts of *anything*, and we will see how to do so with algebraic expressions.

An analogy:

Imagine that you have 3 red rocks and 2 blue rocks. Your friend has 5 red rocks and 1 blue rock. If you combine all the rocks, how many of each type are there altogether?



Altogether, we have 8 red rocks and 3 blue rocks. We <u>do not</u> want to say that "There are 11 rocks altogether", because even though that may be true, *our purpose is to understand how many of each type of rock there are.*

Examples of Polynomial Addition:

We will use the same idea of combining amongst "types of things" for adding polynomials. We will try to figure out how many total constants, variables without an exponent, and variables with an exponent of two, variables with an exponent of three, etc, there are altogether. We will combine constants with constants, combine variables with no exponents with same variables with no exponents, combine the same variables with an exponent of two, and so on.

Example 1: Simplify the following polynomials:

$x^2+2x+5+2x^2+4x+1+3x$

To simplify this, we want to *combine like terms*. Every operation is addition, so we will be adding like terms.

$x^2+2x+5+2x^2+4x+1+3x = 3x^2+9x+6$

Here, each like term is highlighted in its own color, with the terms on the right-hand side of the equals sign showing the answer. When you do this on your own, you may choose to combine like terms one at a time, crossing them off as you go.

Example 2: Simplify the following polynomials:

$(3x^2+4x+7) + (5x^2+6x+1)$

In this instance, we have parentheses around some of the terms, creating two different quantities. These parentheses are used to show that some of the terms are grouped together, separate from others. To solve this, we want to first check what mathematical operations are in the problem. Again, all the operations are addition, so we can look in both groups of parentheses for like terms and add them together.

$(3x^2+4x+7) + (5x^2+6x+1) = 8x^2+10x+8$

Again, each like term is highlighted in its own color, showing how the final answer came to be.

Tip: *Some people find the following advise to be helpful.*

> When you do this on your own, you might combine the x^2 terms first by crossing off the $3x^2$ and $5x^2$ on the left, and writing $8x^2$ on the right side of the equals sign. Then, cross of $4x$ and $6x$ and write $10x$. Finally, cross off the 7 and 1 and write 8. This might help you keep track of the quantities as you combine terms.

Another way to look at the same problem:

$(3x^2+4x+7) + (5x^2+6\text{x}+1) = 3x^2+4x+7 + 5x^2+6\text{x}+1$

We can remove the parentheses between the two group if we are adding two quantities-- this will always be true. From here, we can think about going through the long equation like a list, and "mark off" each part of the equation as we move through adding like terms. In this example, the terms we're combining are highlighted on the left side to show the "marking off", and the right side has the combined terms highlighted to show what they added up to.

So, for $3x^2+4x+7 + 5x^2+6\text{x}+1$ we can have the following steps occur:

$3x^2+4x+7 + 5x^2+6\text{x}+1 = 8x^2 +2x+4+6\text{x}+1$

$8x^2 +4x+7+6\text{x}+1 = 8x^2 +10x+7+1$

$8x^2 +10x+7+1 = 8x^2 +10x+8$

Our final answer of $8x^2 +10x+8$ is the same result as before when we left the parentheses in.

When *adding* polynomials, you have the choice of leaving the parentheses in or removing them-- do what feels best to you. Some people like the parentheses to stay, because it helps them stay organized by looking at like terms in each group of parentheses. Other people find the parentheses distracting and too much, so they look for ways to remove them.

Geometry and Polynomials:

There will be times when you will be asked to solve what looks t*o be both* like a geometry problem *and* polynomial problem. These types of problems can be a little intimidating, partly because it is not set up like a regular geometry *or* polynomial question, and partly because you will have to use your knowledge and skills from two areas together to find the solution.

An example of a geometric polynomial question is below:

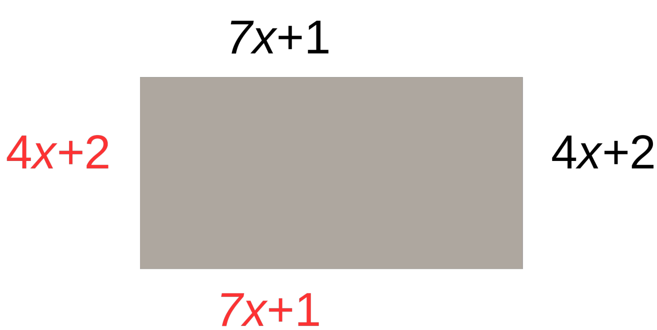
Example 3: Write a polynomial expression describing the perimeter of the rectangle below:

Perimeter: _______________

To solve this problem, consider what it is asking you to do: *write the perimeter.* How do we normally find perimeter, if we imagine the shape having numbers instead of an algebraic expression as side lengths?

Perimeter means to add all the side lengths. Sometimes, there are also special formulas we can use (we could use the formula for perimeter of a rectangle), but we will start with adding all the side lengths.

Notice that not all the side lengths are written for you. This is typical, and you will have to remember that *opposite sides of a rectangle are congruent* (equal to each other). Since opposite sides have the same length, we can write them into the figure. Here, they are written in red.



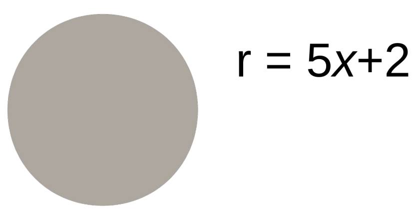
Now that we have all of the side lengths written, and we know that perimeter means add all the side lengths, we can do just that: add like terms.

$7x+1 + 4x+2 + 4x+2 + 7x+1 = 22x+6$

The perimeter of the rectangle is **$22x+6$**

We will now look at an example for using geometry formulas with polynomials.

Example 4: Write an expression describing the circumference of the circle below.



Circumference:___________

This problem wants us to solve for circumference, which is the perimeter of a circle. To do this, we must use the formula for circumference, **$2\pi r$**.

We are given that $r = 5x+2$, so if we substitute that into the formula for circumference, we have the following:

$$2\pi r = 2\pi(5x+2) = 10\pi x+4\pi = 10\cdot 3.14\cdot x+4\cdot 3.14 = 31.4x+12.56$$

Your final answer will be **$31.4x+12.56$**

(Note: we usually use $\pi = 3.14$ for the GED, COMPASS, and other exams. Also, you must distribute in order to solve the problem.)

Word Problems and Polynomials

Sometimes polynomial math questions will be a part of a word problem, and you will have to read carefully to understand what your task is, Consider the following:

Example 5:

> You have $4x^2+5x+1$ dollars in your bank account. On payday, you deposit $7x^2+x+2$ dollars. Write a polynomial expression showing how much money is in your bank account.

Using reasoning skills, figure out what we are asked to do in the problem. The word *deposit* should be the main clue: we are *adding* two quantities together. We have the amount already in the bank, $4x^2+5x+1$, and we will be *adding* $7x^2+x+2$ to it.

$$(4x^2+5x+1) + (7x^2+x+2) = 11x^2 + 6x + 3$$

Polynomial Addition Problem Set

Simplify the following expressions:

$2x^2+8x+1+3x^2+7x+5$

$4x^2+12x+11+2x^2+x+3$

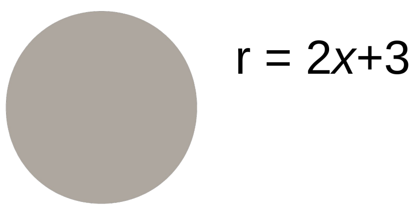
$(3x^2 + 4x + 4) + (3x^2 + 2x + 8)$

$(5x^2 + 2x + 17) + (5x^2 + 4x + 2)$

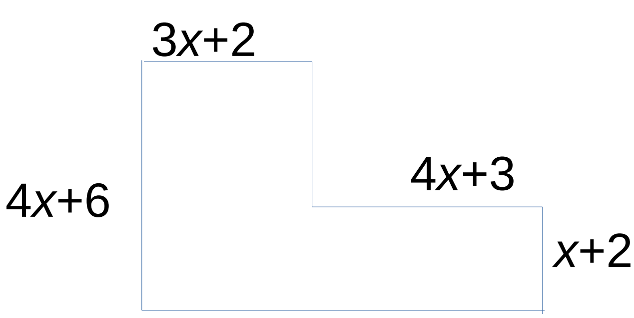
$(4x^2 + 3x + 7) + (x^2 + 2x + 4)$

Given each figure, write an equation for perimeter using the variable *x*.

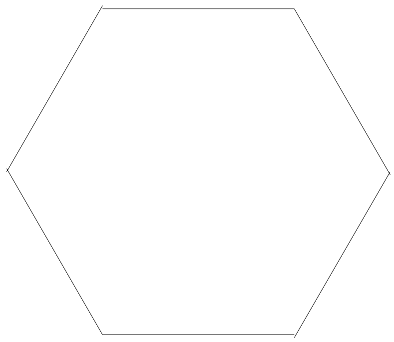
Perimeter: ____________



Circumference:___________



Perimeter:_________



A regular hexagon has side length $3x-4$.

Perimeter: ________________

$2x-4$

Perimeter:_________

$3x+5$

A pentagon has sides that all have a length of 3x. Write a simplified algebraic expression for the perimeter of the pentagon in terms of *x*.

Hint: Draw a picture

A square has an unknown length and width, each represented by *x*. If its length on all sides is increased by 5 units to create a new, larger square, write a simplified algebraic expression for the perimeter of the new square in terms of *x.*

The width of a rectangle is unknown. The length of the rectangle is two more units than its width. Write a simplified algebraic expression for the perimeter of the rectangle in terms of width.

A student simplified the expression $(2x^2+3x+1)+(4x^2+6x+1)$ by adding like this:

$$(2x^2+3x+1) + (4x^2+6x+2) = 15x^2+3$$

Is the student's solution correct? Why or why not?

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