Lesson 4: Multiplying Binomials

One way to multiply two binomial expressions is to arrange both of them in the same way we set up a long multiplication problem. Recall that when we multiply (12)(23), we can solve it by writing the problem like this:

```
      12
*     23
      36     ⇦ the product of 12 and 3
+    24      ⇦ the product of 12 and 2 (usually we put "0" after 24 as a placeholder)
     276     ⇦ the sum of 36 and 240
```

When we solve with long multiplication, we are always careful to line the columns according to place value-- the one's, ten's, hundred's, etc-- so that when we add down the columns, our solution is correct.

To multiply two binomials together, we arrange the problem in the same way as long multiplication, writing the expressions on top of each other. Then, multiply in *the same way as regular long multiplication* following these steps:

- Before starting, remember: you will always keep like terms in the same column: all the constants on the right, x's in the middle, and x^2's on the left.
- First, multiply the top expression and the term furthest on the right together, and write that product down.
- Second, multiply the top expression and the other term together, and write that product below the first product. Keep like terms in the same column together.
- Finally, add the two products together. Notice that you can just add down the columns!

Example 1:

$$\begin{array}{rrrl} & x & +2 & \\ & x & +3 & \\ \hline & +3x & +6 & \Leftarrow \text{ the product of } (x+2) \text{ and } 3 \\ +x^2 & +2x & & \Leftarrow \text{ the product of } (x+2) \text{ and } x \text{ (keep the } 2x \text{ under the } 3x) \\ \hline +x^2 & +5x & +6 & \Leftarrow \text{ the sum of } 3x+6 \text{ and } x^2+2x \text{ is } x^2+5x+6 \end{array}$$

Example 2:

$$\begin{array}{r l}
x\ +4 & \\
x\ +5 & \\
\hline
+5x\ +20 & \Leftarrow \text{the product of } (x+4) \text{ and } 5 \\
+x^2\ +4x & \Leftarrow \text{the product of } (x+4) \text{ and } x \text{ (keep the } 4x \text{ under the } 5x\text{)} \\
\hline
+x^2\ +9x\ +20 & \Leftarrow \text{the sum of } 5x+20 \text{ and } x^2+4x \text{ is } x^2+4x+20
\end{array}$$

Example 3:

$$\begin{array}{r l}
x\ -2 & \\
x\ +3 & \\
\hline
+3x\ -6 & \Leftarrow \text{the product of } (x-2) \text{ and } 3 \\
+x^2\ -2x & \Leftarrow \text{the product of } (x-2) \text{ and } x \text{ (keep the } -2x \text{ under the } 3x\text{)} \\
\hline
+x^2\ +x\ -6 & \Leftarrow \text{the sum of } 3x-6 \text{ and } x^2-2x \text{ is } x^2+x-6
\end{array}$$

Example 4:

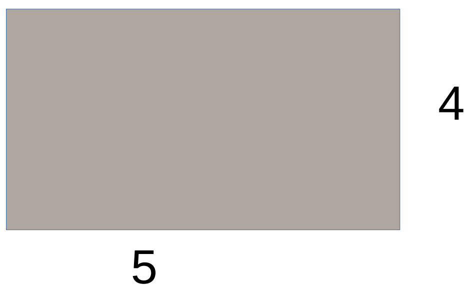
$$\begin{array}{r l}
x\ +3 & \\
x\ -4 & \\
\hline
-4x\ -12 & \Leftarrow \text{the product of } (x+3) \text{ and } -4 \\
+x^2\ +3x & \Leftarrow \text{the product of } (x+3) \text{ and } x \text{ (keep the } 3x \text{ under the } -4x\text{)} \\
\hline
+x^2\ -x\ -12 & \Leftarrow \text{the sum of } -4x-12 \text{ and } x^2+3x \text{ is } x^2-x-12
\end{array}$$

Tips for multiplying binomials:

- The constant in the final product is always the product of the two constants from the original expressions.
- During the last step, we can only add like terms with one another. This is why it is important to write in neat columns and to “skip” a space when multiplying in the second step-- it helps us keep the x's together.

Area Models and Binomial Multiplication

The GED® Test, COMPASS, and other exams will often use *area models* as a way to apply your skills with binomial multiplication. This is because area is always a multiplicative function-- for example, the area of a rectangle is length times width. As a result, you may see an area question on these tests.



In this example, the area of this rectangle is 20, because (5)(4) = 20

In this next example, this rectangle has side lengths (x +2) and (x + 4). However, we still compute area in the same way as we do with integer-length rectangle: multiply length times width, (x +2)(x + 4)

side work to calculate (x +2)(x + 4)

$$\begin{array}{r} x\ +2 \\ x\ +4 \\ \hline +4x\ +8 \\ +x^2 +2x \\ \hline +x^2 +6x\ +8 \end{array}$$

Exercise Set: Binomial Multiplication

Directions: Your task in this assignment is to multiply the pairs of binomials. The first few problems have been set up for you.

$(x+3)(x+4)$

$$\begin{array}{r} x+5 \\ x+3 \\ \hline \end{array}$$

$(x+5)(x+3)$

$$\begin{array}{r} x+4 \\ x-3 \\ \hline \end{array}$$

$(x+4)(x-3)$

$$\begin{array}{r} x+4 \\ X-3 \\ \hline \end{array}$$

$(x+2)(x+4)$

$(x+2)(x+7)$

$(x+4)(x-2)$

$(x-2)(x+5)$

$(x+6)(x-2)$

$(x+3)(x-7)$

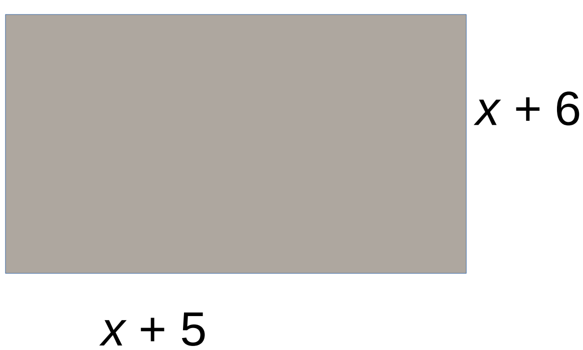
$(x+6)(x-5)$

$(x+6)(x+5)$

$(x+4)(x-4)$

The formula for the area of a triangle is $\text{Area} = \frac{1}{2} \bullet \text{base} \bullet \text{height}$. If the base of a triangle has a length of 8x units, and the height is 4 units, write a simplified algebraic expression for the area of the triangle in terms of x.

Consider the rectangle below. What is its total area?



Does order matter when multiplying binomials? For example, does $(x + 2)(x + 7)$ mean the same as $(x + 7)(x + 2)$?

If the sign between two terms in a binomial is "switched" with another binomial, will multiplying them yield the same result? For example, is $(x + 4)(x – 3)$ the same thing as $(x – 4)(x + 3)$?

Challenge Questions:

Solve the following:

$(x-2)(x-3)$ $(x-4)(x-5)$ $(x-3)^2$